All solutions to graded problems must be typed in LaTeX. All other problems can be typed or written, but must be completed. Graded problems must be organized in numerical order. All problem sets (both graded and non-graded) must be uploaded to Canvas as a single PDF document. Students can find the grading scale in our Syllabus.

Graded

Exercise 1. Let $X = \{a, b, c\}$, and let

$$\mathscr{T}_1 = \{X, \emptyset, \{a\}, \{a, b\}\} \quad \text{and} \quad \mathscr{T}_2 = \{X, \emptyset, \{a\}, \{b, c\}\}.$$

Find the smallest topology containing both $\mathscr{T}_1$ and $\mathscr{T}_2$. Find the largest topology contained in both $\mathscr{T}_1$ and $\mathscr{T}_2$.

Exercise 2. Show that the following collections are bases for the standard topology on $\mathbb{R}$.

1. $\mathscr{B}_1 = \{(x - \frac{1}{n}, x + \frac{1}{n}) : x \in \mathbb{R}, n \in \mathbb{Z}_+\}$
2. $\mathscr{B}_2 = \{(a, b) : a, b \in \mathbb{R} \text{ and } b - a < 1\}$
3. $\mathscr{B}_3 = \{(a, b) : a, b \in \mathbb{Q}\}$

Non-Graded

Exercise 3. Let $\mathscr{B}_l = \{[a, b) : a, b \in \mathbb{R}\}$ and let $\mathbb{R}_l$ denote the *lower limit topology* generated by the basis $\mathscr{B}_l$. Define $\mathscr{C} = \{[a, b) : a, b \in \mathbb{Q}\}$. Show that $\mathscr{C}$ is a basis and show that the topology it generates is distinct from $\mathbb{R}_l$.

Exercise 4. Show that if $\mathscr{A}$ is a basis for a topology on X, then the topology generated by $\mathscr{A}$ equals the intersection of all topologies on X which contain $\mathscr{A}$.